Module Title :

Drives and Traction & Electromagnetic Compatibility

Module Code : ELE 4909

Chapter 1 : Power Semiconductor device

The power semiconductor devices (same as the term Power Electronic Components) are generally used in converters, they can be grouped as follows:

- 1. Diode,
- 2. Thyristors, and
- 3. Power Transistors.

1.1 Diode

A diode is a two-layer p-n semiconductor device. The following figure shows the structure of a diode, its symbol, and V-I characteristics.

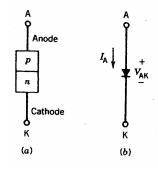
High power diodes are silicon rectifiers that can operate at high junction temperatures.

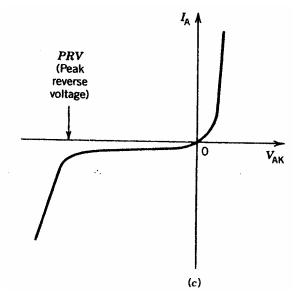
If a reverse voltage is applied across the diode, it behaves essentially as an open circuit.

If a forward voltage is applied, it starts conducting and behaves essentially as a close switch. It can provide uncontrolled ac-to-dc power rectification.

The forward voltage drop when it conducts current is in the range of O.8V to 1.0V.

Diodes with ratings as high as 4000V and 2000A are available.





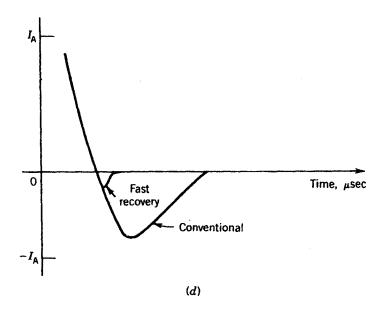
1.1.1 Recovery Characteristics:

Following the end of forward conduction in a diode, a reverse current flows for a short time.

The device does not attain its full blocking capability until the reverse current ceases. The time interval during which reverse current flows is called rectifier recovery time.

During this period, charge carriers stored in the diode at the end of forward conduction are removed. The recovery time is in the range of a few microseconds $(1-5 \sim \mu \text{ sec})$ in a conventional diode to several hundred nanoseconds in fast recovery diodes.

This recovery time is of great significance in high frequency applications. The recovery characteristics of conventional and fast recovery diodes are shown below.



1.2. Thyristors

A thyristor is one of the most important types of power semiconductor devices. Thyristors are used extensively in power electronic circuits. They are operated as bistable switches, operating from non-conducting state to conducting state. Thyristors can be assumed as ideal switches for many applications, but practical thyristors exhibit certain characteristics and limitations.

The thyristor has a 4-layer p-n-p-n structure with 3 terminals, anode (A), cathode (K), and gate (G). The A

and K are connected to the main power circuit. The G terminal carries a low-level gate current in the direction from gate to cathode. The thyristor operates in two stable states (Bistable switch): on or off.

Depending on the physical construction, and turn-on and turn-off behavior, thyristor can broadly be classified into 9 categories:

- 1. Phase-control thyristor (SCR);
- 2. Fast-switching thyristor (SCR);
- 3. Gate-turn-off thyristor (GTO);
- 4. Bi-directional triode thyristor (TRIAC);
- 5. Reverse-conducting thyristor (RCT);
- 6. Static induction thyristor (SITH);
- 7. Light-activated silicon-controlled rectifier (LASCR);
- 8. FET -controlled thyristors (FET -CTH);
- 9. MOS-controlled thyristors (MCT).

1.2.1 SCR

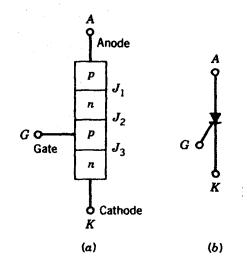
(A) The V-I characteristics of a thyristor (SCR).

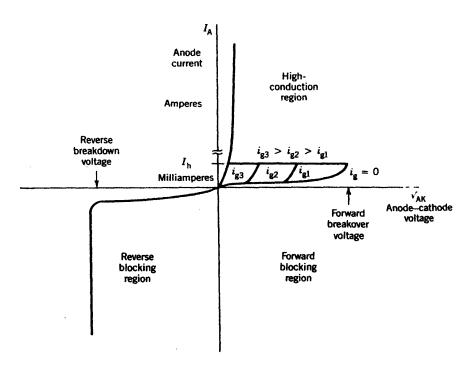
With zero gate current, if a forward voltage is applied across the device (i.e. A is positive w.r.t. K) junction J1 and J3 are forward biased while junction J2 remains reverse biased and therefore the ,mode current is a small leakage current. If the anode-to-cathode forward voltage reaches a critical limit, called the forward breakover voltage, the device switches into high conduction. If gate currents are applied, the forward breakover voltage is reduced. For a sufficiently high gate current, such as i_{g3}, the entire forward blocking region is removed and the device behaves as a diode.

When the device is conducting, the gate current can be removed and the device remains in the on stage. If the anode current falls below a critical limit, called the holding current, the device returns to its forward blocking state.

If a reverse voltage is applied across the device (i.e. anode negative with respect to cathode), the outer

junctions J1 and J3 ,are reverse biased and the central junction J2 is forward biased. Therefore only a small leakage current flows. If the reverse voltage is increased, then at a critical breakdown level (known as the reverse breakdown voltage), an avalanche will occur at J1 and J3 and the current will increase sharply. If this current is not limited to a safe value, power dissipation will increase to a dangerous level that will destroy the device.





(B) Switching Characteristics of a SCR.

If a thyristor is forward biased and a gate pulse is applied, the thyristor switches on.

However, once the thyristor starts conducting an appreciable forward current, the gate has no control on the device. The thyristor will turn off if the anode current becomes zero, called natural commutation, or is forced to become zero, called forced commutation.

However, if a forward voltage is applied immediately after the anode current is reduced to zero, the thyristor will not block the forward voltage and will start conducting again although it is not triggered by a gate pulse. It is therefore necessary to keep the device reverse biased for a finite period before a forward anode voltage can be applied. The period is known as the turn-off time of the thyristor. The turnoff time of the thyristor is defined as the minimum time interval between the instant the anode current becomes zero and the instant the device is capable of blocking the forward voltage.

The switching characteristics of a thyristor are illustrated below figure. The thyristor is turned on by ,l gate pulse ig, which can be obtained from a firing circuit. When the gate pulse i_g is applied at instant t_l , anode current I_A builds up and the voltage across the device (V_{AK}) falls. When the device is fully turned on, the voltage across it is quite small (typically 1 to 2.5V, the higher voltage drop for higher current devices) and

for all practical purposes the device behaves as a short circuit. The device switches on very quickly, the turn-on time ton typically being 1 to 3 μ sec. Typically, the width of the gate pulse is in the range of 10 to 50 μ sec and its amplitude in the range of 20 to 200 mA.

If the current through the thyristor is required to be switched off at a desired instant t_2 , it is momentarily reverse biased by making the cathode positive with respect to the anode (i.e. V_{AK} is negative). For this forced commutation, a commutation circuit is specially designed and required. In most commutation circuits a precharged capacitor is momentarily connected across the conducting thyristor to reverse-bias it.

If the device is reverse biased, its current falls, becomes zero at t_3 , then reverses, and becomes zero again at t_4 . At instant t_5 , the device is capable of blocking a forward voltage. The time interval from t_3 to t_5 is known as the turnoff time of the thyristor.

If a forward voltage appears at instant t_6 the total time interval t_3 to t_6 is known as the circuit turnoff time, t_q .

(B) Practical Consideration.

The circuit turnoff time, t_q , provide to the SCR by the circuit, must be greater than the device turnoff time, t_{off} by a suitable safety margin; otherwise the device will turn on at an undesired instant, a process known as commutation failure.

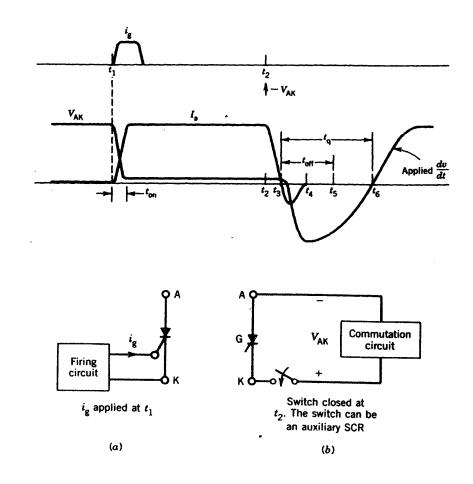
Thyristor having a large turnoff time (50-100 μ sec) are called slow-switching or phase control type thyristors, and those having a small turnoff time (10-50 μ Sec) are called fast switching or inverter type thyristors. In high frequency applications, the required circuit turnoff time becomes an appreciable portion of the total cycle time, and therefore inverter type thyristors must be used.

During the turn-on of the thyristor, if the voltage is high, current is low and vice versa. Therefore, the turn-on switching loss is low. During thyristor turnoff also, if the reverse current is small, the turnoff switching loss is low. The low switching loss in a thyristor is a significant advantage, particularly for high frequency applications.

(D) Protection of a SCR.

If the current in a thyristor rises at too high a rate, i.e. high di/dt, the device can be destroyed. Some inductance must be present or inserted in series with the thyristor so that di/dt is below a safe limit specified by the manufacturer.

A thyristor may turn on (without any gate pulse) if the forward voltage is applied too quickly. This is known as dv/dt turn-on and it may lead to improper operation of the circuit. A simple RC snubber is normally used to limit the dv/dt of the applied forward voltage.

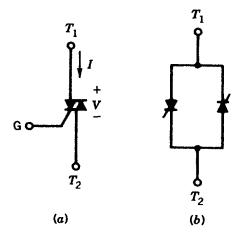


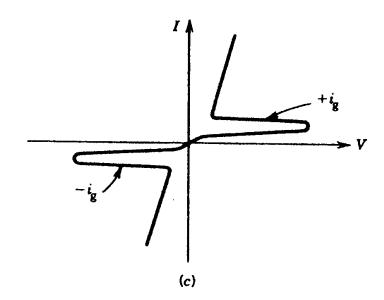
1.2.2 TRIAC

A triac can be considered as an integration of two SCRs in inverse-parallel. The circuit symbol and V-I characteristics of a triac are shown below. When terminal Tl is positive with respect to terminal T2 and the devices is fired by a positive gate current (+ig), it

turns on. Also when terminal T2 is positive with respect to terminal Tl, and the device is fired by a negative gate current (-ig), the device turns on.

A triac is frequently used in many low-power applications such as juice makers, blenders, and vacuum cleaners, etc. It is economical and easy to control compared to two SCRs connected antiparallel. However, a triac has a lower dv/dt capability and a longer turnoff time. It is not available in high voltage and current ratings.





1.2.3 GTO (Gate Turn Off) Thyristor

A GTO thyristor can be turned on by a single pulse of positive gate current (like a thyristor), but in addition it can be turned off by a pulse of negative gate current. Both on-state and off-state operation of the device are therefore controlled by the gate current.

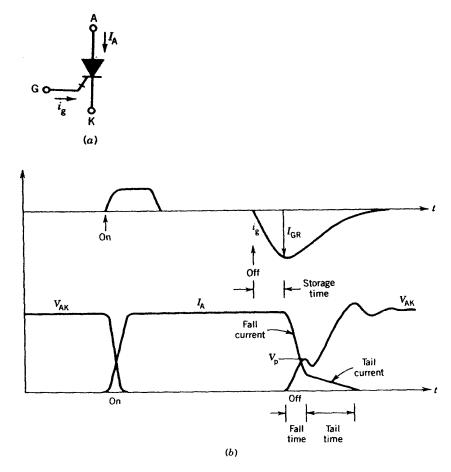
A symbol of GTO used in USA is shown below. The turn-on process is the same as that of a thyristor. The turnoff characteristics are somewhat different. When a negative voltage is applied across the gate and cathode

terminals, the gate current i_g rises. When the gate current reaches its maximum value, I_{GR} , the anode current begins to fall, and the voltage across the device, V_{AK} , begins to rise.

The fall time of I_A is abrupt, typically less than 1 µsec. Thereafter the anode current changes slowly, and this portion of the anode current is known as the tail current. The ratio (I_A/J_{GR}) of the anode current I_A (prior to turnoff) to the maximum negative gate current IGR required for turnoff is low, typically between 3 and 5. For example, a 2500V, 1000A GTO typically requires a peak negative gate current of 250A for turnoff. (IOOOA/250A = 4).

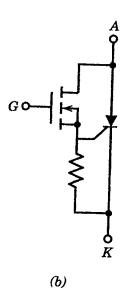
During turnoff both voltage and current are high. Therefore, switching losses are somewhat higher in GTO thyristors. Consequently GTOs are restricted to operate at or below 1kHz switching frequency. If the spike voltage V_p is large, the device may be destroyed. The power losses in the gate drive circuit are also somewhat higher than those of thyristors. However,

since no commutation circuits are required, the overall efficiency of the converter is improved. Elimination of commutation circuits also results in a smaller and less expensive converter.



1.2.4 MCT (MOS-Controlled Thyristor)

The MCT is another hybrid power semiconductor device which combines the attributes of the MOSFET and the thyristor. It has recently become available on the market. The symbol of this device is shown below. The MCT is basically a thyristor which can be turned on and off by a built-in MOSFET type gate. It has the high di/dt (IOOOA/ μ sec), dv/dt (5000V/ μ sec) ratings, and low on-state voltage drop (1 V) and turnoff time (t_{off} = 1.5 μ sec). These superior characteristics make it an ideal power switching device, and thus it has a tremendous potential for use in medium and high power motor drive and power electronic applications.



1.3.1 Power Transistor (BJT)

A transistor is a 3-layer p-n-p or n-p-n semiconductor device having two junctions. This type of tansistor is known as a bipolar junction transistor (BJT).

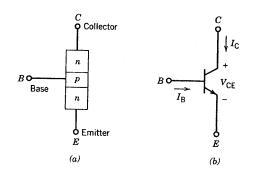
The structure and the symbol of an n-p-n transistor are shown below. The 3-terrninal of the device are called the collector (C), base (B) and emitter (E). The collector and emitter terminals are connected to the main power circuit, and the base terminal is connected to a control signal.

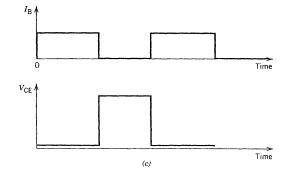
Like thyristors, transistors can also be operated in the switching mode. If the base current I_B is zero, the transistor is in an off state and behaves as an open switch. On the other hand, if the base is driven hard, i.e. if the base current I_B is sufficient to drive the transistor into saturation, then the transistor behaves as a closed switch. The operation is shown below.

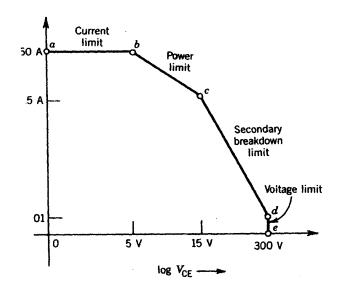
The transistor is a current driven device. The base current determines whether it is in the on-state or the off-state. To keep the device in the on state there must be sufficient base current.

Transistor with high voltage and current ratings are known as Power Transistor. The current gain (Ic/I_{BO} of a power transistor can be as low as 10, even though it is higher than that of a GTO thyristor. High current gain can be obtained from a Darlington connected transistor pair. The pair can be fabricated on one chip, or two discrete transistors can be physically connected as a Darlington transistor. Current gains in the hundreds can be obtained in a high-power Darlington transistor.

Power transistors switch on and switch off much faster than thyristors. They may switch on in less than 1 μ sec and turn off in less than μ sec. Therefore, power transistors can be used in applications where the frequency is as high as 50kHz. These devices are, however, very delicate. They fail under certain high-voltage and high-current conditions. They should be operated within specified limited, known as the safe operating area (SOA).







1.3.2 Power MOSFET (Metal Oxide Semiconductor Field Effect Transistor)

A BJT is a current-controlled device and requires base current for current flow in the collector. Since the collector current is dependent on the input (or base) current, the current gain is highly dependent on the junction temperature.

A power MOSFET is a voltage controlled device and requires only a small input current. The switching speed is very high and the switching times are of the order of nanoseconds.

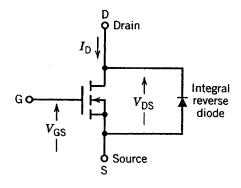
Power MOSFETS are finding increasing applications in low-power high-frequency converters. MOSFETs do not have the problems of second breakdown phenomena as BJTs do. However, MOSFETs have the problems of electrostatic discharge and require special care in handling. In additional, it is relatively difficult to protect them under short-circuited fault conditions. There are other trade names for this devices, such as HEXFET (IR), SIMMOS (Siemens), and TIMOS

HEXFET (IR), SIMMOS (Siemens), and TIMOS (Motorola). The circuit symbol of the MOSFET is shown below. Three terminals are called drain (D), source (S), and gate (0). The current flow is from drain to source. The device has no reverse-voltage blocking capability and it always comes with an integrated reverse rectifier.

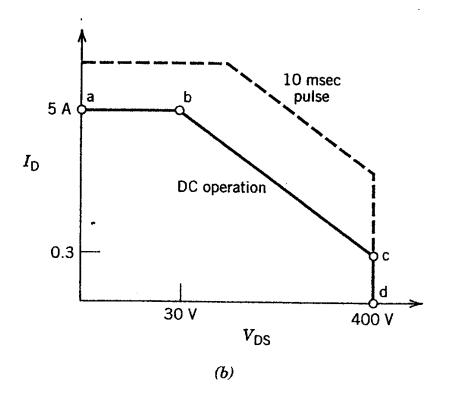
The switching characteristics of the MOSFET are similar to those of the BIT. However, MOSFET switch on and off very fast, in less than 50 nanoseconds. Because MOSFETs call switch under high voltage and current conditions (i.e. practically no secondary

breakdown), no current snubbing is required during turn-off. However, these devices are very sensitive to voltage spikes appearing across them, and snubber circuits may be required t suppress voltage spikes.

MOSFET switch very fast and their switching losses are almost negligible. However, conduction voltage drop is high and therefore conduction loss is high.



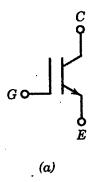
(a)



1.3.3 Insulated Gate Bipolar Transistor (IGBT)

The IGBT is a hybrid power semiconductor device which combines the attributes of the BJT and the MOSFET. It has a MOSFET type gate and therefore has a high input impedance. The gate is voltage driven, as in the MOSFET. The symbol used is shown below. Like the power MOSFET, the IGBT does not exhibit the secondary breakdown phenomenon, common to the BJT. As well, the IGBT has low on-state voltage drop, similar to the BJT. The switching speed of the IGBT is significantly lower than the MOSFET and is similar to the BJT.

IGBTs are available with ratings of 1500V, 1000A, and they are presently preferred to BJTs.



Comparison of High Power Devices

	SCR	GTO	BJT	MOSFET	IGBT	MCT
Voltage (V)	4000	4000	1000	600	1500	600
Current (A)	3000	3000	500	50	1000	75
Frequency (kHz)	1	2	30	1000	30	10
On-state voltage drop (V)	1.5-2.5	1.5-2.5	2-3	3-4	2-3	1
di/dt (A/μsec)	500	500				1000
dv/dt (V/μsec)	500	500				5000
T _{off} (μsec)	10-100	5-10	<2	<0.1	<2	1.5

Chapter 2 : Electric Drive

2.1 A Definition

An electric drive is an industrial system which performs the conversion of electrical energy to mechanical energy (in motoring) or vice versa (in generator braking) for running various processes such as: production plants, transportation of people or goods, home appliances, pumps, air compressors, computer disc drives, robots, music or image players, etc.

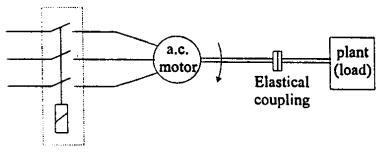
About 50% of electrical energy produced is used in electric drives today. Electric drives may run at constant speed (Figure 1.1) or at variable speed (Figure 1.2).

The constant speed electric drive contains the electric (alternating current) motor, the mechanical coupling, the mechanical load (plant) and the electromechanical (or electronic) start/stop and protection system. Today about (75-80)% of all electric drives still run at constant speed because there is not

much need for speed control except for starting, stopping and protection.

However there is a smaller (20-25)% group of applications, with very fast annual expansion rate where the torque and speed must be varied to match the mechanical load.

Traditionally for variable speed d.c. brush motors have been used for decades but a.c. motors have been catching up lately (since 1990) as shown below. This radical shift is mainly due to the rapid progress in power electronic converters for a.c. motors.



Electromechanical (or electronic) start/stop and protection system

Figure 1.1. Constant speed electric drive

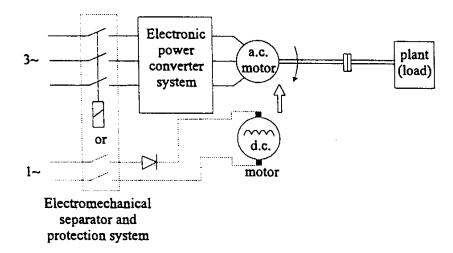
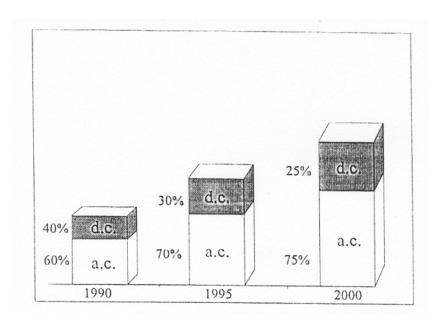
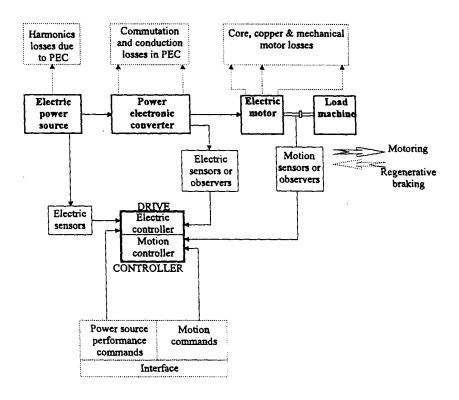


Figure 1.2. Variable speed electric drive



A modem electric drive capable of controlled variable speed, is made of some important parts such as:

- > the electric motor;
- > the power electronic converter (PEC);
- > the electric and motion sensors;
- > the drive controller;
- > the command interface.



Torque-Speed Characteristics of D.C. Machine

For the separately-excited d.c. machine, the flux is assumed to be constant,

Therefore,

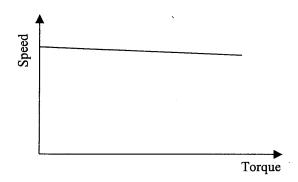
$$\tau = KI_a$$
 & $E = K\omega$

And the circuit equation gives,

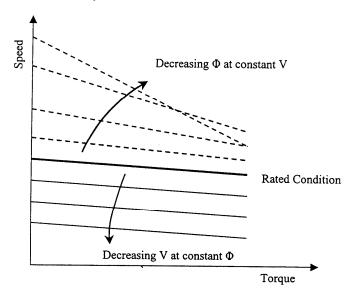
$$V = E + I_a R_a$$

$$\omega = \frac{V}{K} - \frac{R_a}{K^2} \tau$$

This equation shows the torque-speed characteristic as follow:



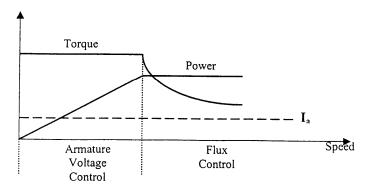
The torque-speed curves by both control is shown as follow:



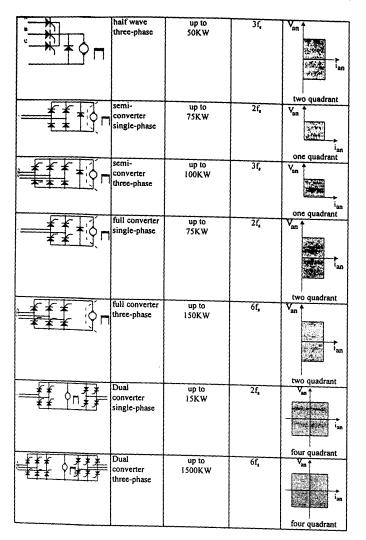
From the figure, it can be seen that the characteristic remains the same for armature voltage control. Therefore, the maximum torque capability is unchanged for different value of terminal voltage. And this region is best for constant torque speed control.

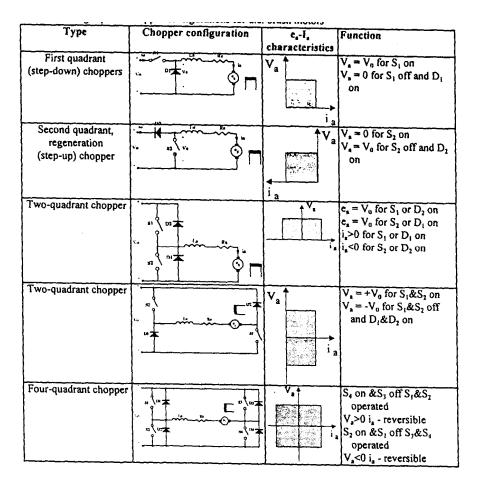
For the flux control, the maximum torque capability gets deteriorated when the flux is decreased. However, the armature current could still be held at a maximum value as rated value. Together with a constant V, the E will remains constant with constant I_a , as a result, the output power = EI_a is also constant. The flux control is best for constant power control.

Combined constant torque and power control of the same Ia is shown below:



Circuit type		Power range	Ripple frequency	Quadrant operation
	half wave single-phase	below 0.5KW	f _s	Van ian one quadrant





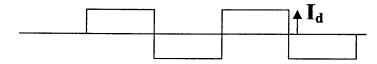
The general solution of power factor is given as follow:

$$p.f. = \frac{V_{dc \text{ output}} \times I_{dc \text{ output}}}{V_{rms \text{ input}} \times I_{rms \text{ input}}}$$

For 1-phase fully controlled rectifier,

$$p.f. = \frac{\frac{2V_m}{\pi} \cos \alpha \times I_d}{\frac{V_m}{\sqrt{2}} \times I_d}$$

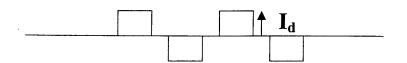
where I_{rms} of a.c. input = I_d



For 3-phase fully controlled rectifier,

$$p.f. = \frac{\frac{3V_m}{\pi} \cos \alpha \times I_d}{\sqrt{3} \frac{V_m}{\sqrt{2}} \times \sqrt{\frac{2}{3}} I_d}$$

where I_{rms} of a.c. input = $\sqrt{\frac{2}{3}}I_d$



2.2 DC Motor Control

2.2.1 Multiquadrant Operation

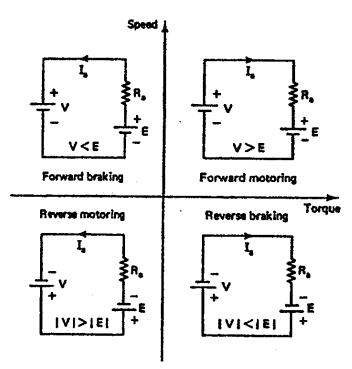
Multiquadrant drives employing semiconductor converters mostly employ regenerative braking because of saving in energy. Figure below shows the polarities of the source voltage, back emf, and armature current for the operation in different quadrants.

D.C. motor operates with positive speed and torque in quadrant I. The power flows from source to motor.

In quadrant II, the motor experiences the regenerative braking, i.e. the power flow motor to source.

When the motor gets stopped in quadrant II, it is ready to motor in reverse direction in quadrant III. Finally, the motor experiences the regenerative braking again in quadrant IV.

The direction of the source voltage and the armature current is expressed as follow in terms of different quadrant.



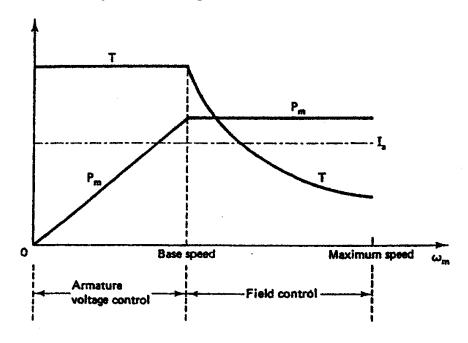
	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
Voltage	+	. +	-	-
Ia	+	-	· -	+

The direction of the armature current is in line with the motor torque $T\alpha \phi Ia$. It should be noted that the reversal is not possible for the rectifier which can only carry current in one direction.

The control of D.C. motor speed under the base speed with the converter control of the armature

voltage could achieve a constant torque from zero to base speed of the motor.

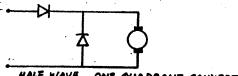
The Speed achieved over the base speed shall be obtained by controlling the motor field.



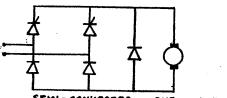
2.2 AC/DC Converter

The phase controlled rectifier is classified as the half-controlled and fully controlled type. They could achieve the motor operation in certain quadrants.

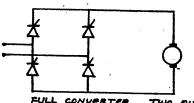
SINGLE PHASE THYRISTOR PHASE CONTROLLED CONVERTERS



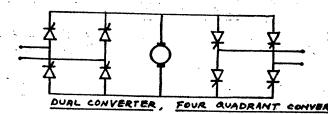
HALF WAVE , ONE QUADRANT CONVERTER



SEMI-CONVERTER, ONE QUADRANT CONVERTER



FULL CONVERTER, TWO QUADRANT CONVERTER



2.3 Ratings of Converter and Motors

During transient operations such as starting, braking, speed reversal, speed changing, and so on, the motor current can be allowed to be higher than its rated current, due to its large thermal capacity.

Converters use semiconductor elements which do not have any capacity for overload, due to their low thermal capacity. Therefore, their current rating is chosen as equal to the maximum current that may be required to flow through the motor.

2.4 Harmonics Effect

All power semiconductor converters have harmonics in their output voltage and current. The harmonic currents increase the rms value of the motor current and distort flux. Consequently, the copper and core losses are increased. To prevent the motor temperature from exceeding a safe value, the load on the motor must be less than rated. In other words, the motor has to be derated in the presence of harmonics.

2.5 Advantages of Converters

It has high efficiency, fast response, control flexibility, easy maintenance, reliability, low weight and volume, less noise, long life, and so on.

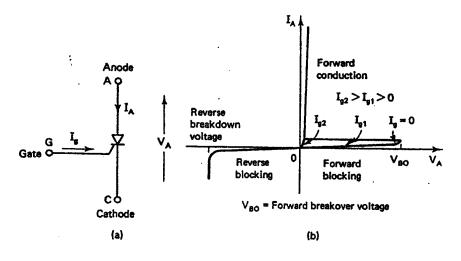
2.6 Power Semiconductor Devices

2.6.1 Thyristors

In the absence of the gate current I_g , the current carried by the thyristor for the anode to cathode voltage V_A , less than the forward breakover voltage V_{BO} and greater than the reverse breakdown voltage (relative value), is close to zero.

After turn-on, when I_A reaches a value known as the latching current, the thyristor continues to conduct ven after the gate signal has been removed. Hence only a pulse of current is required for turn-on.

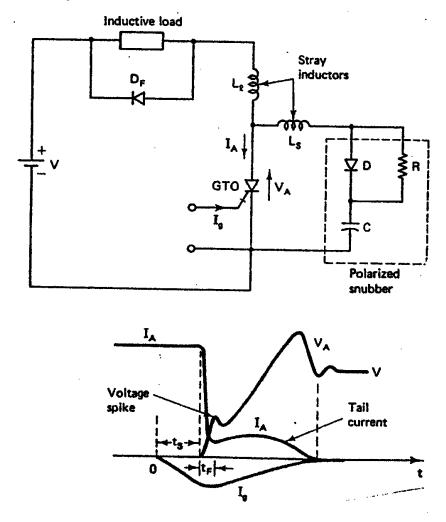
The limitation of the thyristor is that it cannot be turned off by the gate signal. For turn-off - also known as commutation - anode current, IA must be reduced below a value which is known as the holding current. Subsequently, it should be subjected to a reverse bias of sufficient duration for it to regain forward voltage blocking capability.



2.6.2 Gate- Turn-Off Thyristors

GTO is a special thyristor where turn-off can be achieved by a negative gate

current pulse. GTO turn-off circuit has much lower loss than thyristor commutation circuit.



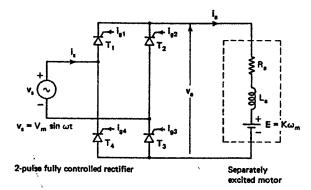
2.7 Single-Phase Fully-Controlled Rectifier

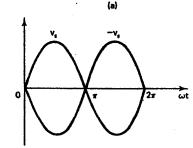
Figure below, shows a 1-phase fully-controlled rectifier supplying a dc separately excited motor. The armature has been replaced by its equivalent circuit. R_a and L_a are the armature circuit resistance and inductance respectively, and E is the back emf.

The thyristor pair TI, T3 receives firing pulses from α to π and the pair T2, T4 receives firing pulses from $(\pi + \alpha)$ to 2π .

2.7.1 Modes of Operation

The drive operates either as motoring mode or regenerative braking mode. And it is said to operate in discontinuous conduction when i_a becomes zero for a finite interval of time in each cycle.







 V_m = the peak value of the supply voltage, V_m

 ω = supply frequency, rad/sec.

 β = angle at which the armature current drops to zero value, rad

 $\beta' = \beta - \pi$

 $\gamma = \sin^{-1}(E/V_m)$, at which $V_s = E$

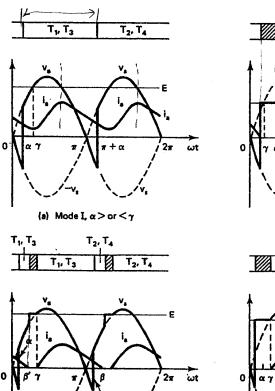
 $\gamma' = \pi - \sin^{-1}(|E|/V_m)$

2.7.1.1 Motoring

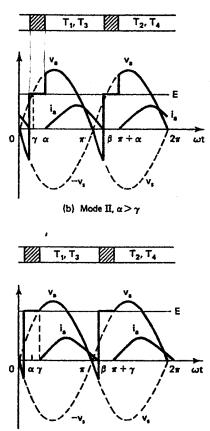
When armature current flows through the source, and either through the pair T_1 , T_3 or through the pair T_2 , T_4 . When the pair T_1 , T_3 conducts, $V_a = V_s$, and when the pair T_2 , T_4 conducts. $V_a = -V_s$. When none of the thyristor pair conducts, $i_a = 0$ and $V_a = E$.

When $i_a > 0$ at the instant of the firing a thyristor pair, then the biasing on thyristors of this pair will be decided by the source voltage only. If the source voltage provides a positive bias, thyristors will turn on even when the source voltage is less than E.

When i_a =0 at the instant of firing a thyristor pair, then the biasing on thyristors of this pair will be decided by the difference of the source voltage and the back emf. The thyristors of this pair will conduct if the source voltage has the appropriate polarity and its magnitude is higher than E.



(c) Mode III, $\alpha < \gamma$



(d) Mode IV, $\alpha < \gamma$

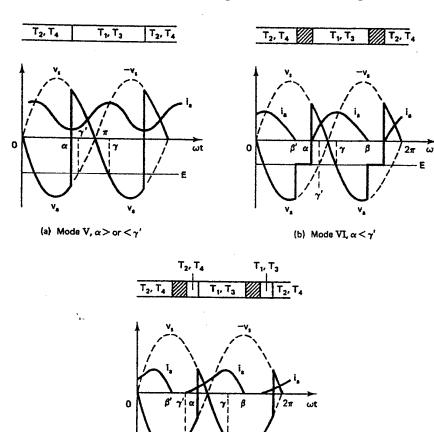
2.7.1.2 Regenerative Braking

To provide regenerative braking operation, the system must be able to feed back the power from the motor side to the supply side. The reverse of current flow would not be possible as rectifier can carry current only in one direction. The only alternative available for the reversal of the flow of power is to reverse both the rectifier output voltage V_a and the motor back emf E with respect to the rectifier terminals and make $|E| > V_a$.

In single-phase controlled rectifier, the output voltage can be reversed by making $\alpha > 90^{\circ}$ And the reversal of back emf E is made possible by

- a) By coupling an active load to the motor shaft in the reverse direction, this gives reverse regeneration (quadrant IV), no terminals change is required.
- b) By reversing the field current, this gives forward regeneration (quadrant II). And no changes in armature terminals is required.

c) By reversing the armature connections with respect the rectifier terminals, this gives forward regeneration.



2.7.2 Speed-Torque Characteristics

Under steady state at the continuous armature current operation, the circuit equation is given as follows:

$$V_a = I_a R_a + K \omega_m$$

$$I_{a} = \frac{(2V_{m}/\pi)\cos\alpha - K\omega_{m}}{R_{a}}$$

Where $\tau = KI_{a: and} K = K_{motor}\Phi$, the relationship between speed and torque is as follows:

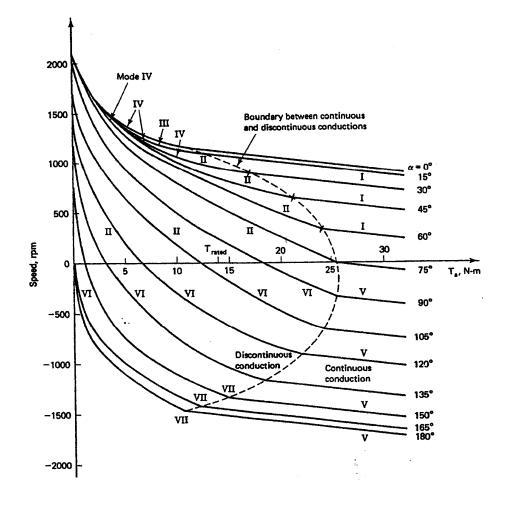
$$\omega_{\rm m} = \frac{2v_{\rm m}}{\pi K} \cos\alpha - \frac{R_{\rm a}}{K^2} \tau_{\rm a}$$

The speed-torque curves belong to a 2.2kW, 1500 rpm dc motor led by 1-phase controller. The boundary between continuous and discontinuous conduction is shown by a dotted line. For torques less than the rated value, a low-power drive operates predominantly in the discontinuous conduction. In continuous conduction, the speed-torque characteristics are parallel straight

(c) Mode VII, $\alpha > \gamma'$

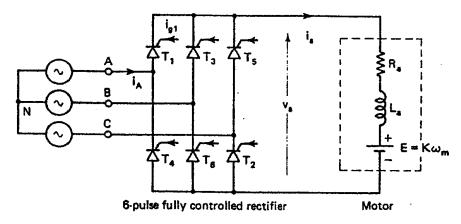
lines, whose slope, according to equation, depends on the armature circuit resistance Ra. The effect of discontinuous conduction is to make the speed regulation poor. Other disadvantages of discontinuous conduction are the nonlinear transfer characteristics of the converter and (the slower transient response of the drive.

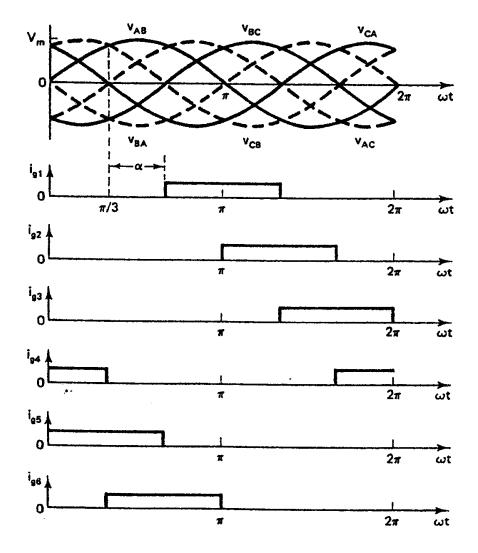
The filter inductor could be added to the armature circuit to increase the effective impedance angle to reduce the zone of discontinuous conduction. However, the addition of the filter inductance increases the losses, armature circuit time constant, noise, and cost, weight, and volume of the drive.



2.8 3-phase Fully-controlled Rectifier

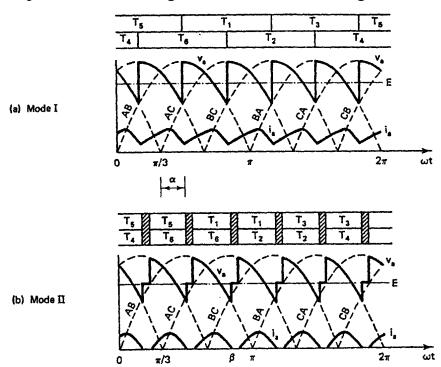
The most widely used dc drive is the three-phase fully-controlled bridge rectifier fed dc separately excited motor drive shown in the figure. Thyristors are fired in the sequence they are numbered, with a phase difference of 60°. The line commutation of an odd-numbered thyristor occurs with the turning on of the next odd-numbered thyristors. The same is true for even-numbered thyristors. Consequently. each thyristor conducts for 120° and only two thyristors conduct at a time - one odd-numbered and one even-numbered.

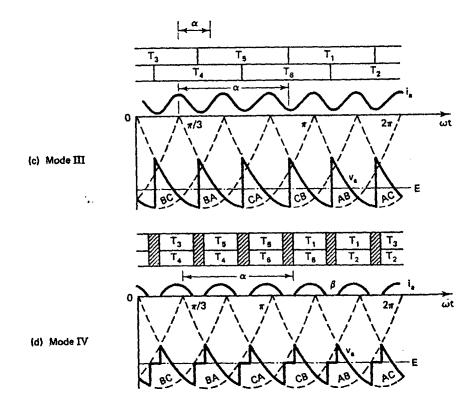




2.8.1 Modes of Operations

The transfer of current from an outgoing to incoming thyristor can take place when the respective line voltage is of such a polarity that not only does it forward bias the incoming thyristor but it also leads to reverse biasing of the outgoing thyristor when the incoming thyristor turns on. Thus, the firing angle for a thyristor is measured from the instant when the respective line voltage is zero and increasing.





For continuous current operation in mode I, the converter output voltage from

$$\omega t = \alpha + \pi / 3 to \alpha + 2 \pi / 3$$

$$V_a = \frac{3}{\pi} \int_{\alpha + \frac{\pi}{3}}^{\alpha + \frac{2\pi}{3}} Vm \sin \omega t d(\omega t) = \frac{3}{\pi} V_m \cos \alpha = V_{ao} \cos \alpha$$

Where
$$V_{ao}=3V_m/\pi$$
,

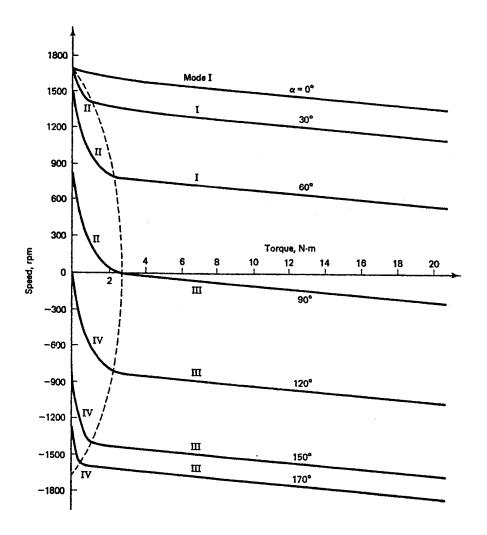
2.8.2 Speed torque Characteristics

The speed torque characteristics of a 2.2kW, 1500 rpm uc motor fed by a 3-phase (fully-controlled rectifier with an ac source voltage of $170.3 \ V_{line}$, 50Hz is shown in the figure. The regions of continuous and discontinuous conductions and the modes of operation have been marked. Comparison of these characteristics with those of figure shows a considerable reduction in the zone of discontinuous conduction.

The ideal no-load operation is obtained when $E = V_m$ for $0 \le \alpha \le \pi/6$ rad. and $E = V_m \sin(\alpha + \pi/3)$ for $\pi/6 \le \alpha \le \pi$ radians. Thus, the no-load speeds are given by the following equations:

$$\omega_{mo} = \frac{V_m}{K} \qquad , 0 \le \alpha \le \pi/6$$

$$\omega_{mo} = \frac{V_m \sin(\alpha + \pi/3)}{K}$$
 , $\pi/6 \le \alpha \le \pi$



For mode I operation, the continuous current motoring operation gives the relationship between speed and torque is

$$\omega_m = \frac{3V_m}{\pi K} \cos \alpha - \frac{R_a}{K^2} T_a$$

For mode II operation, we have the initial condition $i_a(\alpha + \pi/3) = 0$ for the equation:

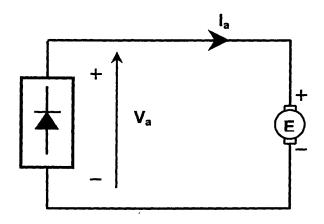
$$V_a = L_a \frac{di_a}{dt} + R_a i_a + K\omega_m = V_m \sin \omega t, \qquad \left(\alpha + \frac{\pi}{3}\right) \le \omega t \le \beta$$

As a result, the speed-torque equation is solved as follows:

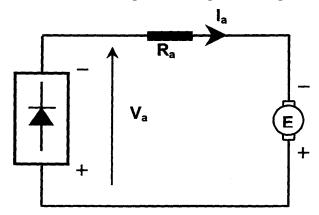
$$\omega_m = \frac{V_m \{\cos(\alpha + \pi/3) - \cos\beta\}}{K(\beta - \alpha - \pi/3)} - \frac{\pi R_a}{3K^2(\beta - \alpha - \pi/3)}T_a$$

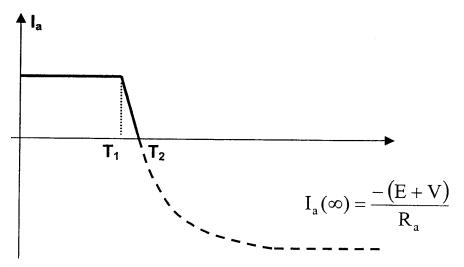
2.9 Advanced Firing Scheme (Transition from Quadrant I to II)

Quadrant I: Motoring



Quadrant II: Regenerating Braking





At T_1 , $\alpha = Maximum(180^\circ)$, current is forced to zero at high rate

At T₂, firing pulses are stopped, and emf(armature) reversal or field reversal is carried out

After reversal, firing pulses are released with the same maximum firing angle to prevent sudden rise of current

 $\boldsymbol{\alpha}$ is gradually decreased for a desired armature current

2.10 Power Factor Development

For the calculation of the rectifier power factor, it is customary to assume that the supply voltage is sinusoidal. In a dc drive, the rectifier power factor and the source current harmonics depend on speed, torque, and firing angle. Assume the load inductance is big enough to maintain a constant load current flowing and the rectifier loss is neglected. The general solution of power factor is given as follow:

$$p.f. = \frac{V_{dc \ output} \times I_{dc \ output}}{V_{rms \ input} \times I_{rms \ input}}$$

For 1-phase fully controlled rectifier

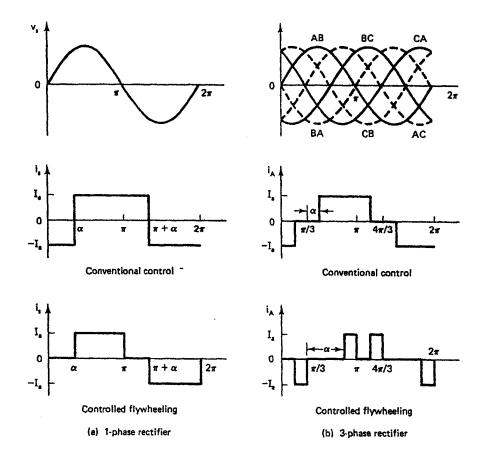
$$p.f. = \frac{\frac{2V_m}{\pi} \cos \alpha \times I_d}{\frac{V_m}{\sqrt{2}} \times I_d}$$

where I_{rms} of a.c. input = I_d

For 3-phase fully controlled rectifier

$$p.f. = \frac{\frac{2V_m}{\pi} \cos \alpha \times I_d}{\frac{V_m}{\sqrt{2}} \times I_d}$$
where I_{rms} of a.c. input = $\sqrt{\frac{2}{3}}I_d$

For the comparison of various rectifiers, it is common to assume a ripple-less armature current. With this assumption, the source current wavefom1s for some rectifiers are shown as follow.



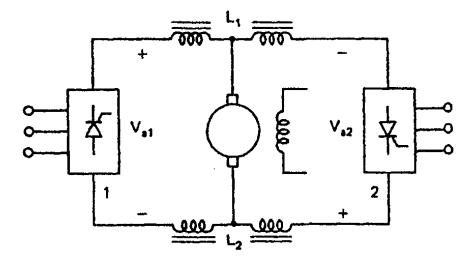
The expressions for the power factor, fundamental power factor, and nth harmonic expressed as a ratio of the dc component and per-unit output voltage are listed below:

	Control Parameters	P.F.	Fundamental F.F.		In/Ia
1-phase rectifier	0≤α≤π	0.9cosα	cosα	$\frac{0.9}{n}$	n=odd
3-phase rectifier	0≤α≤π	0.955cosα	cosα	0.78 n	n=1,5,7,

2.11 Multiquadrant Operation by Dual

Converter

A dual converter consists of two fully-controlled rectifiers connected in antipallel across motor armature shown in the figure. If rectifier 1 provides operation in the first and fourth quadrants, rectifier 2 provides operation in the second and third quadrants.



The dual converter may operate in simultaneous or non-simultaneous control.

2.11.1 Non-simultaneous Control

Also know as circulating-current-free control, only one rectifier operates, at any given time and another is blocked. And the speed reversal is carried as follow:

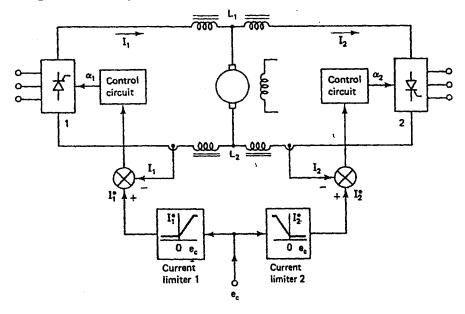
- a) Rectifier 1 must initially operate in the second quadrant and then in the third quadrant.
- b) Rectifier 1 must be turned off before rectifier 2 could operate in quadrant III, otherwise, a line side short-circuit between them will take place.
- c) The armature current is forced to zero by setting the firing angle of rectifier 1.
- d) After the zero current is sensed, a dead time of 2 to 10 ms is provided to ensure the turn-off of all the thyristors of rectifier 1.
- e) Then the firing pulse for rectifier I is withdrawn and released to rectifier 2.

f) The motor speed will not change appreciably during this period owing to inertia.

Because of the delay involved in zero current, it is not usually employed in high performance drives.

2.11.2 Simultaneous Control

The dual converter for simultaneous control is shown below, the two rectifiers are controlled simultaneously in such a manner that the Sum of their average terminal voltages is zero so that no dc current circulates in the loop formed by the two rectifiers.



The operation equations are:

$$V_{a1} + V_{a2} = 0$$

$$V_{a0}\cos\alpha_1 + V_{a0}\cos\alpha_2 = 0$$

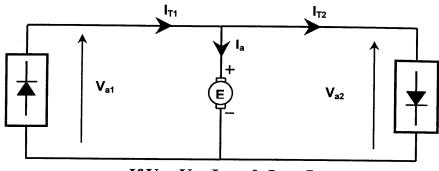
$$\cos\alpha_1 = -\cos\alpha_2$$

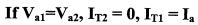
$$\alpha_1 + \alpha_2 = 180^{\circ}$$

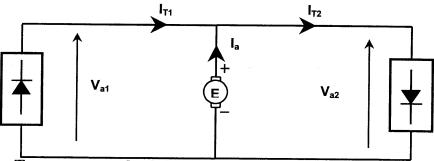
The equation shows that when one rectifier rectifies another inverts. To reverse the direction,

- a) α_1 is increased and α_2 is decreased.
- b) At the moment of motor back exceeds $|V_{a1}|$ and $|V_{a2}|$, armature current shifts to rectifier 2 and operates in second quadrant.
- c) Further increase of α 2, results in deceleration under regenerative braking.
- d) At about $\alpha_{1} \approx \alpha_{2} \approx 90^{\circ}$, zero speed is obtained.
- e) Further reduction of α 2 below 90° motor will accelerate in reverse.

Dual Converter (simultaneous)



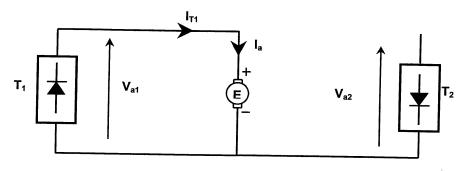




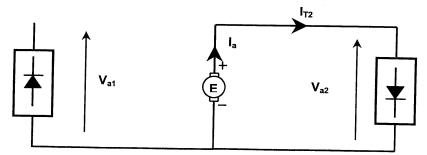
To regenerate, both V_{a1} & V_{a2} is reduced to make a reverse flow of I_a

- $\quad I_{T2} = I_a$
- $-I_{T1}=0$

Dual Converter (non-simultaneous)

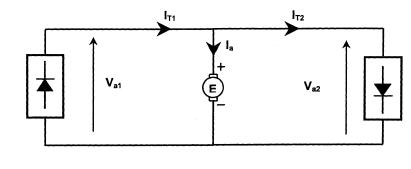


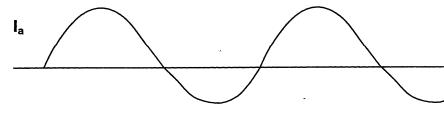
 $\alpha_1 = \text{max. to make } I_a = 0,$

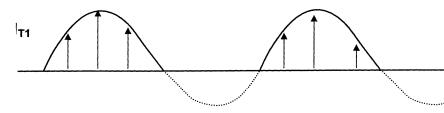


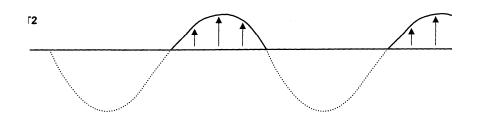
When $I_a = 0$, T_1 is OFF, T_2 is ON with $\alpha_2 = max$. Then, α_2 is reduced to allow reverse flow of I_a

Simultaneous Dual Converter Continuous Conduction at Light Load









2.11.3 Comparison of Non-simultaneous and simultaneous controls

Non-simultaneous Control:

- 1. There is no circulating current. The power factor is high.
- 2. There is no reactor. The response is quick.
- 3. Discontinuous conduction at light load leads non-linear transfer characteristics
- 4. There are control zone. It needs the detection of current zero during armature reversal.

Simultaneous Control:

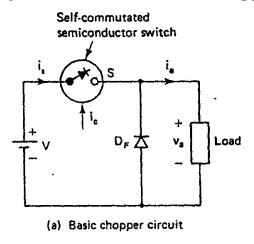
- 1. It allows Bi-directional flow of I_a. It gives continuous conduction (even at light load).
- 2. Transfer characteristics are linear. It provides a good speed regulation.
- 3. There is additional power loss due to the circulating current.
- 4. Transfer response and power factor are low as it needs reactors to limit circulating current.

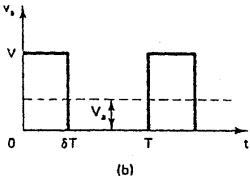
2.12 Chopper Control

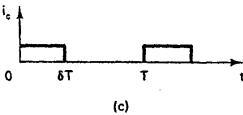
Chopper control could achieve a higher frequency of the ripple and therefore, the discontinuous conduction area is smaller. The speed regulation and the transient response is therefore improved.

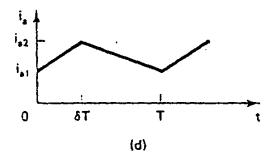
2.12.1 Step-Down Chopper(class A)

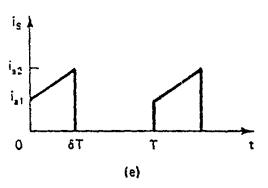
The semiconductor switch S is operated periodically with a period T and remains closed for a time t_{on} = δT with O< δ <1. The variable δ (= t_{on} /T) is called the duty ratio or duty cycle of a chopper. The control signal ic will be a base current for a transistor chopper, and a gate current for the GTO chopper or the thyristor.











During the on interval of the switch, $(0 \le t \le \delta T)$, the load is subjected to a voltage V and the load current increases (from i_{al} and i_{a2} . The switch is opened at $t=\delta T$. During the off period of the switch, $(\delta T \le t \le T)$, the load inductance maintains the flow of current through diode DF. The load terminal voltage stays zero and the current decreases from i_{a2} to i_{al} . The interval $\delta T \le t \le T$ is called the duty interval and the interval $\delta T \le t \le T$ is known as the freewheeling interval. Diode D_F provides a path for the load current to flow when switch S is off and thus improves the load current waveform.

The average component or average value of the load V_a is given by

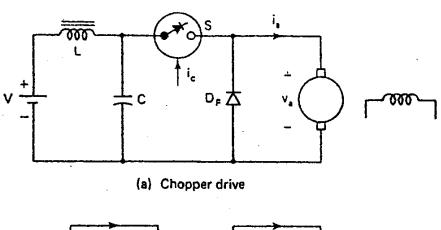
$$V_a = \frac{1}{T} \int_0^T v_a dt = \frac{1}{T} \int_0^{T} V dt = \delta V$$

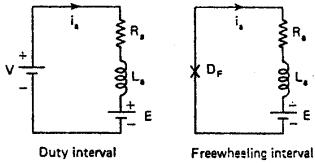
By controlling δ between 0 and 1, a chopper allows a variable dc voltage to be obtained from a fixed voltage dc source. The source current waveform consists of a fundamental ac harmonics which frequency is the same as the chopper frequency. At higher chopper frequencies, harmonics can be reduced to a tolerable level by a cheaper filter. The ripple in the load current decreases as the chopping frequency is increased or the load inductance is increased.

With present day semiconductor devices, choppers operate at frequencies which are sufficiently high to eliminate discontinuous conduction during the motor's normal steady-state operation. Discontinuous conduction takes place only under the transient operation. In view of this, steady-state analysis will be considered only for the continuous conduction.

2.12.2 Steady-State Analysis of Step-down Chopper

The analysis aims at the calculation of the armature current ripple and the torque-speed relationship for a separately excited control motor.





(b) Equivalent circuits

Duty Interval($0 \le t \le \delta T$): figure (b)

$$R_a i_a + L_a \frac{di_a}{dt} + E = V \qquad (i)$$

Let $i_a(0)=i_{a1}$

Solution of equation (i) with this initial condition is

$$i_a = \left(\frac{V - E}{R_a}\right) (1 - e^{-t/\tau_a}) + i_{a1} e^{-t/\tau_a}$$
 (ii)

Where $\tau_a = L_a/R_a$, the armature circuit time constant. If the current at the end of the duty interval is i_{a2} , then from equation (ii),

$$i_{a2} = \frac{V - E}{R_a} \left(1 - e^{-\delta T + \tau_a} \right) + i_{a1} e^{-\delta T + \tau_a}$$

Freewheeling Interval($\delta \le t \le T$),

$$R_a i_a + L_a \frac{di_a}{dt} + E = 0 \qquad (iii)$$

Where $t' = t - \delta T$.

The initial current (at t' = 0) is ia 2. Solving equation (iii) with this initial condition gives

$$i_a = -\frac{E}{R_-} (1 - e^{-t'/\tau_a}) + i_{a2} e^{-t'/\tau_a}$$

In the steady state, the value of i_a at the end of the chopping cycle should be the same as at the beginning of the cycle. Thus, the value of i_a for $t'=(1-\delta)T$ will be ia1. Substituting this in equation gives

$$i_{a1} = -\frac{E}{R_a} \left(1 - e^{-(1-\delta)T/\tau_a} \right) + i_{a2} e^{-(1-\delta)T/\tau_a}$$
 (iv)

Solving for ial & ia2 gives,

$$i_{a1} = \frac{V}{R_a} \left(\frac{e^{\delta T/\tau_a} - 1}{e^{T/\tau_a} - 1} \right) - \frac{E}{R_a}$$

$$i_{a2} = \frac{V}{R_a} \left(\frac{1 - e^{-\delta T/\tau_a}}{1 - e^{-T/\tau_a}} \right) - \frac{E}{R_a}$$

The current ripple Δi_a is given by the following equation:

$$\Delta i_a = \frac{i_{a2} - i_{a1}}{2} = \frac{V}{2R_a} \left[\frac{1 + e^{T/\tau_a} - e^{\delta T/\tau_a} - e^{(1-\delta)T/\tau_a}}{e^{T/\tau_a} - 1} \right]$$

Now, the torque speed relationship could be determined from the steady state circuit equation:

$$V_a = E + R_a I_a$$

 $\delta V = E + I_a R_a$

or

$$I_a = \frac{\delta V - E}{R_a}$$

Since the flux is constant for separately-excited motor and the ac components of armature current would produce ac torque which would have a zero average value. Therefore, the motor torque is given by

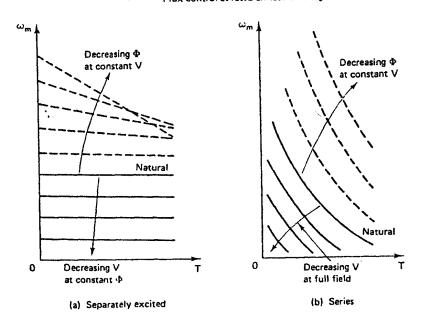
$$T_a = KI_a$$

Substituting the torque equation into the armature current equation gives

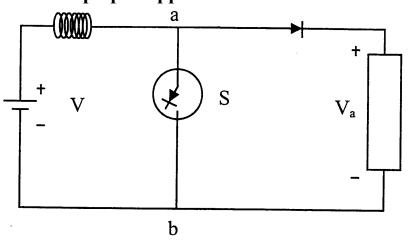
$$\omega_m = \frac{\delta V}{K} - \frac{R_a}{K^2} T_a$$

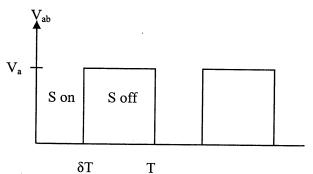
To achieve speed above the rated speed, the field control would be required by reducing the field current by a chopper while the armature shall be directly connected to the supply. The duty ratio of the field chopper is reduced to get higher speed.

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2.12.3 Step up chopper





$$V_{L} = \frac{1}{T} \int_{0}^{T} \left(L \frac{di}{dt} \right) dt$$

$$=\frac{1}{T}\int_{i_{0}}^{i_{0}}Ldi=0$$

$$V_{ab} = (1-\delta)V_a$$

$$V = V_L + V_{ab}$$

$$V_L = 0$$

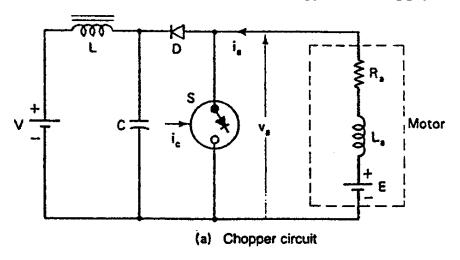
$$V_a = \frac{V}{1 - \delta}$$

According to equation, theoretically the output voltage V_a can be changed from V to ∞ by controlling δ from 0 to 1. In practice V_a can be controlled from V to a higher voltage, which depends on C, and the parameters of the load and chopper.

The main advantage of a step-up chopper is the low ripple in the source current. It is widely applied in low-power battery-driven vehicles such as golf carts, trolleys and so on. The principle of the step-chopper is also used in the regenerative braking of dc motors.

2.12.4 Regenerative Braking of DC Motors

The regenerative braking circuit is shown below. It uses essentially the step-up(class B) chopper to step-up the lower back emf to feedback energy to the supply.



The armature inductance now serves as a magnetic storage during the ON period of the switch ($O \le t \le \delta T$) when the armature terminal would be short-circuited. As a result, the armature current would be increased from i_{sl} to i_{s2} . The mechanical energy supplied by the load and the inertia of the motor load system (only if the speed is charging) is converted into electrical energy.

During the OFF period of the switch, the sum of the energy stored in the armature inductance and the machine energy would be transferred to the source.

Energy Storage Interval ($O \le t \le \delta T$)

The switch is closed, and the armature terminals are short-circuited.

$$R_a i_a + L_a \frac{di_a}{dt} = E \qquad i_a(0) = i_{a1}$$

Energy Transfer Interval ($\delta T \le t \le T$)

The switch is open, and

$$R_a i_a + L_a \frac{di_a}{dt} + V = E$$

$$i_a(\delta T) = i_{a2}$$

The machine is working as a generator,

$$V_a = E - I_a R_a$$

$$V_a = (1 - \delta)V$$

$$(1 - \delta)V = E - I_a R_a$$

$$I_{u} = \frac{E - (1 - \delta)V}{R_{u}}$$

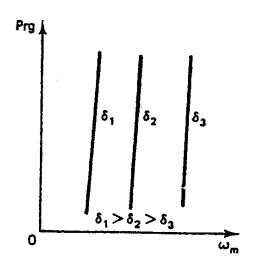
The regenerative power P_{rg} is given by

$$P_{rg} = \frac{1}{T} \int_{0}^{(1-\delta)T} (V \times i_a) dt$$

By solving the i_a in both the energy storage and energy transfer interval gives,

$$P_{ry} = \frac{V^2}{R_a} \left[\left(\frac{E}{V} - 1 \right) \bullet \left(1 - \delta \right) + \frac{\tau_a}{T} \left\{ \frac{e^{(1-\delta)T/\tau_a} + e^{\delta T/\tau_a} - e^{T/\tau_a} - 1}{1 - e^{T/\tau_a}} \right\} \right]$$

The braking power against the speed is shown below. For a given δ , the regenerative power increases linearly with speed.



To maintain constant torque braking, or constant armature current in the braking, we could assume i_a is kept at a desire value.

$$E = (1 - \delta)V + I_a R_a$$

$$K\phi\omega = (1 - \omega)V + I_aR_a$$

Since, \varnothing is constant for separately-excited or for series motor $(I_a = I_f)$,

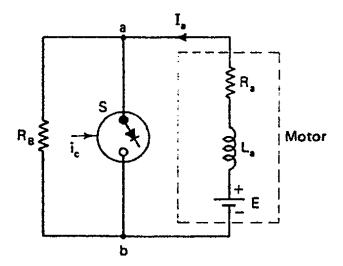
$$K \omega = (1 - \delta)V + I_a R_a$$

The maximum speed would be obtained at 0 = 0, or some practical minimum value. The minimum speed would be obtained at 0 = 1. The regenerative braking would only be effective between these two speeds.

If however, at δ =0 or a minimum value, the mechanical speed is higher than the maximum speed, possibly caused by an active load, then I_a would be increased beyond the original desired value. Since no further reduction of 0 is possible, the armature current has to be reduced by inserting additional armature resistance or by weakening the field to reduce the emf so as to limit the armature current.

2.12.5 Dynamic Braking

The switch is connected in parallel with the braking resistance R_B . The circuit allows a stepless variation in the effective value of the resistance between the terminals a, b.



Neglecting the armature current ripple, the energy consumed by the resistance R_{B} in one cycle of the chopper is given by

$$E_{\rm B} = I_{\rm a2}R_{\rm B}(1-\delta)T$$

The average power consumed by $R_{\mbox{\scriptsize B}}$

$$P_a = \frac{E_B}{T} = I_a^2 R_B (1 - \delta)$$

The effective value of the resistance between the terminals a, b

$$R_e = \frac{P_a}{I_a^2} = (1 - \delta)R_B$$

For dynamic braking in series motor, it must be self-excited. Then the field coil has to be reversed to correspond to the reverse of the field/armature current.

To regenerative brake the motor, the circuit equation is given as

$$K\omega = (1 - \delta)V + I_a R_a$$

If the motor is braked at high speed, the value of δ has to be reduced to balance the equation.

- 1. The higher the ω , the smaller the δ .
- 2. Until δ is minimum, ω is the maximum speed for braking.

3. Further increase of ω , I_a will be increased or just by reducing flux to reduce the effective value of emf.

$$(1-\delta)V = K\omega - I_a R_a$$

$$I_a = \frac{K\omega - (1 - \delta)V}{R_a}$$

Braking current can be controlled by δ .

Now, if the motor is braked at high speed,

 $K\omega \rightarrow increase$

 $I_a \rightarrow increase$

 $\delta \rightarrow$ should be decrease to reduce I_a .

Method of determining the maximum speed for braking:

Critical speed:

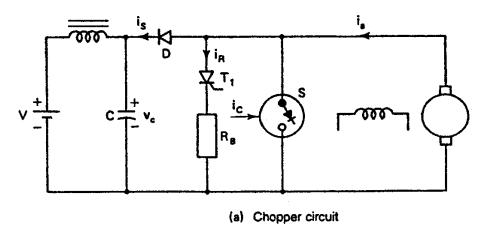
- 1. Higher speed at braking demands lower value of δ .
- 2. When δ reaches its minimum, no more increase in speed is allowed, or I_a exceeds desired value.

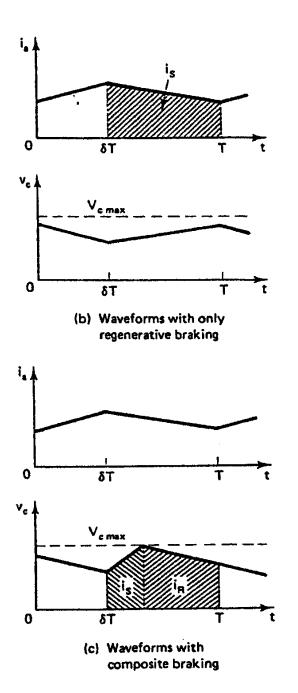
- 3. Speed at the minimum value of δ is called critical speed.
- 4. When δ is minimum and speed is higher than critical speed, I_a can be reduced by either increase R_a or decrease the flux.

2.12.6 Composite Braking

When the regenerative braking is in effect, the energy would be stored in the source or supplied to the load connected to the source (reception). If the source is an a.c. supply, the regenerative energy has to be transferred back to source by a line commutated inverter; this would increase the equipment costs.

The energy absorption can be by other loads such as other traction vehicles on the line. If however, no such load is available at the time of braking and no inverter is installed, then the voltage would be built up rapidly to exceed a safe value.





Then, an external resistor R_B could be inserted into the circuit to absorb the energy. This is so called rheostatic braking with the circuit shown. During the regenerative braking($\delta T \le t \le T$), the voltage would build up if the network is not receptive. Until it reaches the maximum allowable voltage, the thyristor Tl is turned on so that all current will flow through R_B instead of to the source through diode D. The combination of this resistor in the regenerative braking is called composite braking.

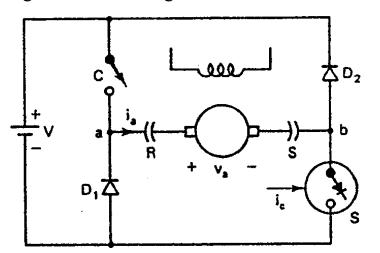
Its advantages are the flexibility of energy flow without violation of the maximum safe voltage and the simplicity of the control circuit.

2.12.7 Two-quadrant Control of Chopper-Fed DC Motors

Two-quadrant operation consisting of forward and regenerative braking requires a chopper capable of giving a positive voltage and current in either direction. This two-quadrant operation of dc motors can be realized in two ways.

Single Chopper with a Reversing Switch

The chopper circuits for forward motoring and forward regenerative braking are combined as below.



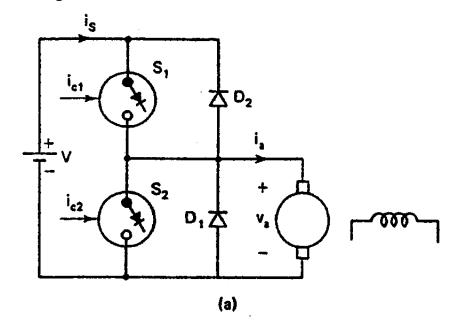
S is a self—commutated semiconductor switch which is closed for δT and the remaining period OFF. When C, a switch, is closed, the circuit permits forward motoring. When the switch C is open and the armature terminals are reversed, the circuit functions for regenerative braking.

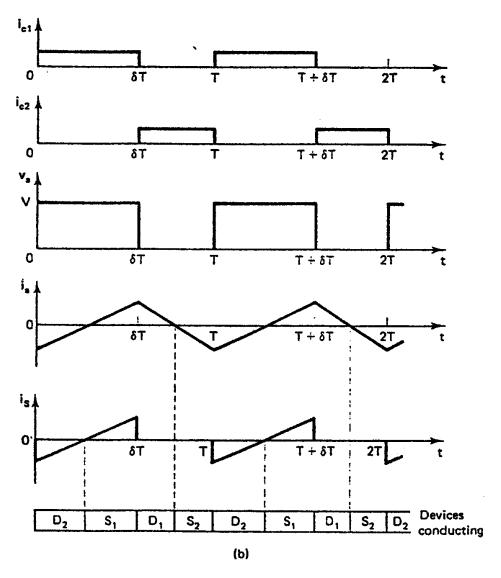
The changeover from motoring to braking happens by first deactivate the switch S and open the switch C. Then the back emf and the supply voltage act in the same direction. This would force the reversal of the armature current. However, once the current is reduced to zero, it would not flow in reverse due to D2 and D1. After an adequate delay to ensure the current has indeed become zero, the armature connections are reversed and switch S is reactivated with a suitable value of 0 to start regeneration.

The chopper can also be used for series motor provided that the field connection is outside the reversing switch to ensure the field direction could be maintained the same for both forward motoring and forward braking. The major difference is the armature emf would be reduced to a very low value just before the changeover because the field or armature is zero. The build-up of the armature emf after changeover would rely on the residual magnetism.

Class C Two-Quadrant Chopper

In some applications, such as servo drives, machine tools, and so on, a smooth transition from motoring to braking and vice versa is required. Class C chopper is required for them. It provides quadrant I & II applications. While two semiconductor switches are required, it does not require any other switches to change the circuit.





Forward motoring and braking control using class C two quadrant chopper: (a) Chopper circuit, (b) Waveforms.

The circuit operation is as follow:

- a) The switches S_1 & S_2 are closed alternately. When S_1 is off ($\delta T \le t \le T$), the supply is disconnected to the machine, the armature current has to flow through D_1 ; flywheeling interval
- b) During the flywheeling interval, if the armature current i_a is dropped to zero. The positive emf would drive a current flowing in reverse through S_2 .
- c) In $(0 \le t \le \delta T)$, the armature current may flow through the S_1 or D_2 as long as emf is present; energy transfer interval. If the current flows from supply to motor via S_1 , the energy transfer is from source to motor. If the current flows from armature terminal to supply, the energy is transferred from machine to supply.
- d) Since the circuit allows both directions of armature current flow in each interval, the conduction is essentially a continuous one.
- e) In motoring, only S₁, and D₂ would conduct. In forward regenerative braking, only S₂ and D₁ would conduct. In a very light load/no load condition, both

four devices would conduct in their way.

f) Whether it be motoring or regenerative braking, the armature ripple would be driven either higher or lower below zero to make the average current positive or negative. In light load, the average current may be smaller than the ripple and resulting in four devices to conduct.

From the waveform, the terminal voltage is given by $V_a = \delta V$ Hence, $I_a = (\delta V - E)/R_a$

The equation suggests that the motoring operation (+ve I_a) takes place when δ >(E/V), and that regenerative braking (-ve I_a) occurs when δ <(E/V). The no load operation is obtained when δ =(E/V).